World Transactions on Engineering and Technology Education
Vol.11, No.2, 2013

Using a miniature water fountain made in the laboratory to illustrate how to apply the energy equation to internal flows through tubes

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ABSTRACT: What was being learned in an introductory course in fluid mechanics was applied to the analysis and testing of miniature water fountains. Different fountains had been designed and constructed by different groups of students for the purpose of applying what they learned to a real-life situation. Each fountain consisted of two jars that were arranged in such a way that one rested on top of the other. The lower jar was open to the atmosphere, while the upper jar was so hermetically sealed so that only water could enter and leave it during operation. Once pressure in the upper jar was reduced to that below atmospheric, water could be lifted from the lower to the upper jar by means of a length of plastic tubing and continuous flow could thus be sustained by a siphon effect. First analyses and experiments were carried out for steady flows and the two sets of results were compared; then, analyses and experiments were carried out again but for unsteady flows and the two sets of results were also compared; the effects of unsteadiness were identified. It was found that analyses underestimated the effects of losses and those of unsteadiness on the flows tested.

INTRODUCTION

Many courses in the engineering curriculum are both mathematically intensive and challenging to students. Fluid mechanics is one of them. It is very important that students not only understand the material taught in these courses, but also that they be able to apply it to the solution of practical problems; after all, this is what most students will be doing after they graduate and go on to the world of engineering practice.

This article gives a brief report on a hands-on project that was used in fluid mechanics lectures to achieve this purpose. This project was carried out by small groups of students who worked together.

There is a large body of research and classroom experience that supports the importance and the effectiveness of hands-on learning as a supplement to lectures. This effectiveness is based on the facts that students have different learning styles in general [1-7]; that engineering students and faculty have different learning styles [9-13]; that collaborative work enhances learning [14]; and that experiential learning increases the depth of understanding [15-17].

THE PROJECT

The project consists of designing, constructing, analysing and testing the performance of a miniature water fountain made from two jars and two pieces of plastic tubing. The ultimate aim is to compare the predictions of analysis with the results of experiments.

Miniature water fountains can be built using jars and pieces of plastic tubing such as straws as shown in Figure 1. They can be constructed using the step-by-step instructions that are described below:

a. Find two transparent jars (say, A and B) with good lids and two pieces of plastic tubing of different lengths; for example, if one cannot find a long piece of tubing, then, create a longer straw by connecting three regular straws end to end and hold the resulting assembly together with adhesive tape.
b. Punch two holes in the lid of one of the jars (say, jar A). Note that empty jars that once contained jam, mayonnaise, jelly, canned fruit, etc, have been used successfully. The other jar (B) will be a reservoir that is open to the atmosphere.
c. Take jar (A) and push one end of the short plastic tube a distance of two inches (5 cm) through one of the holes and seal that joint tightly after insertion.
d. Push the longer straw/tube through the other hole and stop pushing it shortly after the inserted end of the straw can be clearly seen through the wall of jar (A). Seal this joint tightly as well. At this point, find a way to put a cap onto

the free end of the long straw. This cap will make it possible to open and close the end of the long straw as needed during experiments. Use the cap to close that end.

e. Pour enough water into jar (A) to cover the end of the long straw without allowing water to exit that jar through the short straw. Screw the lid to jar (A) after that.
f. Fill the open jar (B) with water, turn the *empty* jar (A) upside down, and let the free end of the short straw dip into the jar that is full of water. Check that the setup is complete and sealed adequately.
g. Find a way to hold the setup securely in place on an elevated platform, so you can run experiments.
h. Place a graduated cylinder under the capped end of the long straw to collect the water that will be exiting the upper jar during testing.
i. If sealing has been adequate, then, when the free end of the long straw is opened, a jet of water will rise from the upper end of the short straw into the upper jar (A), and it will keep flowing until the jar that was filled with water (supply jar) is empty or until the free surface of water in the intake jar falls below the reach of the straw. It is very important to check for leaks and to make sure that your setup will allow you to collect the data that are needed for the study.

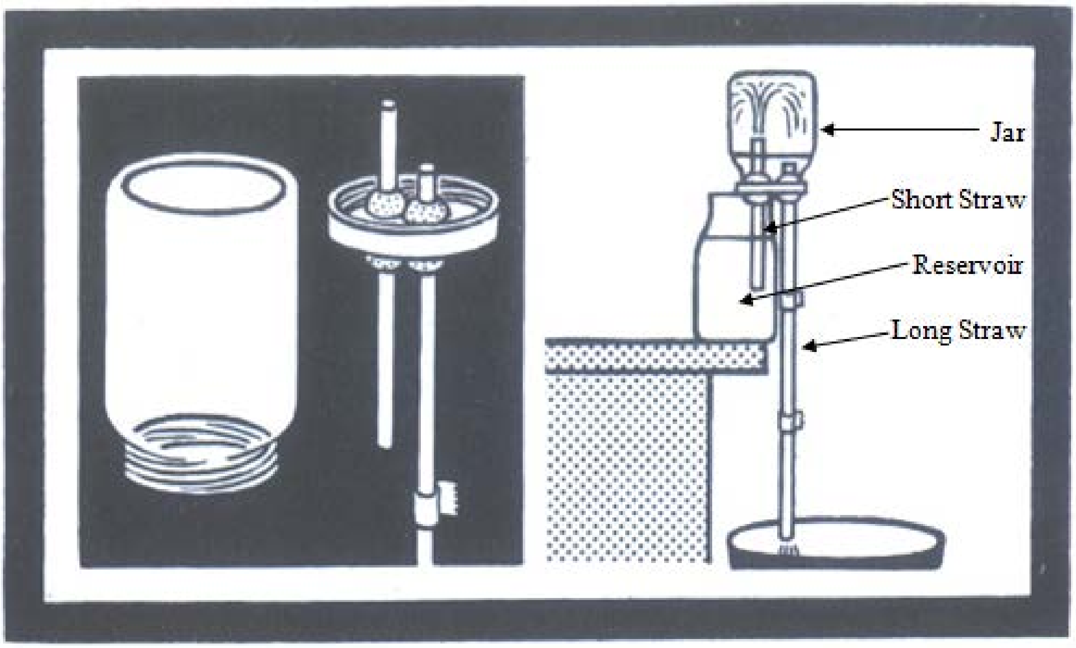

Figure 1: Sketch of a miniature water fountain built and used as a testing apparatus [18].

The setup shown in Figure 1 is that of a functioning miniature fountain; its performance was analysed using the energy equation that was being taught in the fluid mechanics lectures; it was also tested experimentally. Then, the results from analysis and those obtained from testing were compared.

Two types of flow are investigated: steady flow and unsteady flow of water produced by the fountain. In each type of flow, the aim is to use analysis to predict how the volume flow rate of water produced by the fountain varies with the depth of submersion of the short straw into the water contained in the intake reservoir.

Similarly, in each type of flow, experiments were carried out to measure the actual volume flow rates produced by the fountain using different depths of submersion of the short straw into the intake reservoir. Flow rates generated in steady flows were also compared with those generated in unsteady flows in order to estimate the effect of unsteadiness.

The remainder of the article is organised in the following way. First, the mechanical behaviour of the fountain is analysed using the energy equation for the flow of a viscous and incompressible fluid as applied to an inertial control volume. This is done for both steady flows and unsteady flows. Then, experiments that were carried out with the fountain operating under steady conditions are discussed, and their results are presented and compared. After that, experiments that were carried out with the fountain operating under unsteady conditions are discussed, and their results are presented and compared as well.

Finally, the effects of unsteadiness on volume flow rates produced by the fountain are discussed by comparing the results obtained from steady and unsteady experiments.

In the model shown in Figure 2, L_L is the length of the long straw (used for exit); L_S is the length of the short straw (used for intake); and Z_2 is the elevation of the free surface of water in the intake reservoir above a datum. In this figure, the lower end of the long straw is used as the datum.

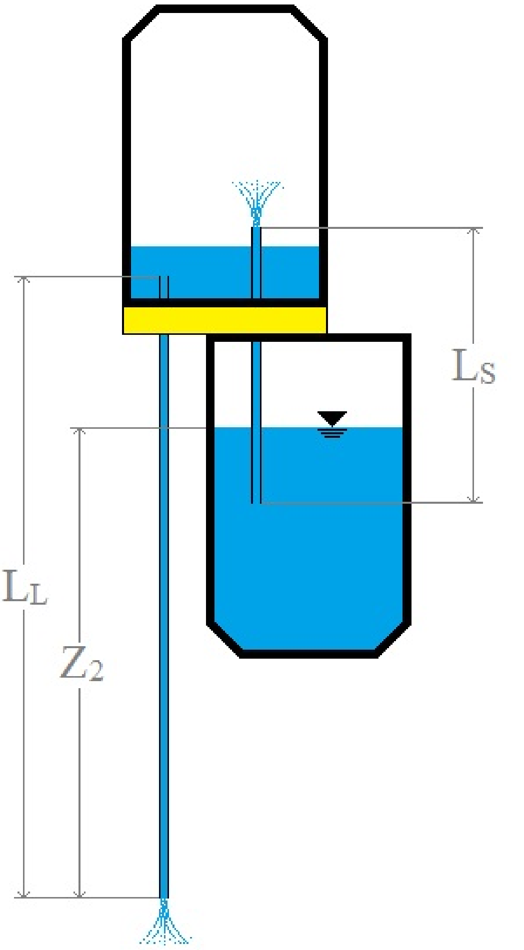

Figure 2: Experimental apparatus used (model to the left; built apparatus to the right) [19].

MODEL AND ANALYSIS OF THE FLOW PRODUCED BY THE FOUNTAIN

Consider two points, 1 and 2, along the same streamline that is followed by a given particle in a flow along a pipe, where the fluid is incompressible and the flow is steady. The energy equation along that streamline is given by Equation (1), where p is the static pressure, V is the speed of the fluid particle, g is the acceleration of gravity, z is the elevation of the location of the fluid particle relative to a datum, and α is the kinetic energy coefficient. Total head losses are divided into two: major losses, h_l, due to friction, and minor losses, h_{lm}, representing all other losses [20].

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{V_1^{\,2}}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{V_2^{\,2}}{2}+gz_2\right)=head\ loss\ total=h_l+h_{lm} \qquad (1)$$

The h_l and h_{lm} can be found as follows:

$$h_l=f\frac{L}{D}\frac{V^2}{2} \qquad\qquad h_{lm}=K\frac{V_2^2}{2}$$

Where f is the friction factor, L is the length of the straw, D the diameter of the straw, and K the loss coefficient. If both straws have the same diameter, then, $L = L_S + L_L$. For the case of steady flow through the fountain being analysed here, choosing where to place points 1 and 2 along the streamline can introduce simplifications. Let point 1 be located on the free surface of the intake reservoir.

Steady flow is achieved by keeping the elevation of this free surface constant during the operation of the fountain. Hence, $V_1 = 0$ and $p_1 = p_{atmospheric}$. Similarly, let point 2 be located at the exit from the long pipe, where the water that leaves the fountain issues into the atmosphere. Hence, $p_2 = p_{atmospheric}$. Major losses occur along the long straw, as well as along the short straw.

Minor losses occur when the water enters the short straw and when it enters the long straw. Minor losses also occur as water emerges from the short straw and splashes onto the free surface of the water inside the upper jar. It can be seen from Figure 2 that both straws are re-entrant tubes, for which K = 0.78. If the datum is set at the lower end of the long straw, then, as shown in Figure 2, Z_2 is the difference between the elevations of points 1 and 2.

Solving for V_2 from Equation (1), one obtains:

$$V_2 = \sqrt{\frac{2g(z_1 - z_2)}{\left(f\frac{L}{D} + 2K + \alpha_2\right)}} \tag{2}$$

The volume flow rate delivered during operation of the fountain is:

$$Q_2 = \left(\frac{\pi D^2}{2}\right) V_2$$

In other words, using Equation (2), Q_2 becomes:

$$Q_2 = \left(\frac{\pi D^2}{2}\right) \sqrt{\frac{2g(z_1 - z_2)}{\left(f\frac{L}{D} + 2K + \alpha_2\right)}} \tag{3}$$

It is expected that the change in the Reynolds number during the operation of the fountain will be very small, which allows one to expect the friction factor f to remain an essential constant during experiments. It follows that the volume flow rate is proportional to the square root of the quantity (z_1-z_2), the difference between the elevations of points 1 and 2. From Figure 2, it can be seen that (z_1-z_2) consists of the sum of the depth of submersion of the short straw into water and a fixed fraction of the length of the long straw.

Since the latter is fixed in a given design, the only part of (z_1-z_2) that can vary during testing is the depth of submersion of the short straw. It follows that Equation (3) gives the variation of the volume flow rate produced by the fountain as a function of the depth of submersion of the short straw. The friction factor will be determined once the Reynolds number of the flow and the relative roughness of the straw are known; the loss coefficient for re-entrant pipes is obtained from published Tables [20], and the kinetic energy coefficient, α_2, depends on the flow regime: for turbulent flows $\alpha_2 = 1$; and for laminar flows, it is 2 approximately [20].

EXPERIMENTAL VALIDATION: THE CASE OF STEADY FLOWS

For the flows investigated, the values for the long straw used in the laboratory were between 50 cm and 80 cm; those for the short straws were between 18 cm and 30 cm; the diameters of the straws were between 0.5 and 0.6 cm; the relative roughness was around 0.0003; the Reynolds' numbers were between 2,000 and 5,000; and the friction factors varied between 0.025 and 0.04. Representative results obtained from testing are shown below in Figures 3 and 4.

The fountain was tested under steady conditions. Here, both the level of the free surface of water in the intake reservoir and the depth of submersion of the short straw were kept constant during a given trial. The level of water in the reservoir was kept constant during the operation of the fountain by adding water to it at the rate that was required to do so. After each trial, the depth of submersion was set to a new value by increasing it by a constant increment; then, the experiment was repeated.

This was repeated as many times as the sizes of the reservoir and short straw would allow. In the data reported below, ten depths of submersion were tested using 1 cm increments. At each depth, 200 ml of water were collected and the time taken to collect each sample volume was recorded. Thereafter, the corresponding flow rates were calculated by dividing each volume of fluid collected by the recorded time. The results for steady flows are plotted in Figure 3, where the dimensions of the straws used were D = 0.6 cm, L_S = 19 cm, and L_L = 56.5 cm.

EXPERIMENTAL VALIDATION: THE CASE OF UNSTEADY FLOWS

For the case of unsteady flow, the energy equation will be used with the added effect of the local acceleration of the fluid particles along the streamline. In this case, Equation (1) is modified to become Equation (4), shown below, where the term:

$$\int_1^2 \frac{\partial V}{\partial t} ds,$$

represents the energy expended in accelerating the fluid particle along its streamline.

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + g z_2\right) = h_l + h_{lm} + \int_1^2 \frac{\partial V}{\partial t} ds \tag{4}$$

The analytical evaluation of the term:

$$\int_1^2 \frac{\partial V}{\partial t} ds,$$

is difficult, because the variation of the local acceleration along the streamline is not known a priori. It is for this reason that this term is often neglected in examples given in fluid mechanics textbooks. However, this term can be estimated experimentally, which was done in these experiments.

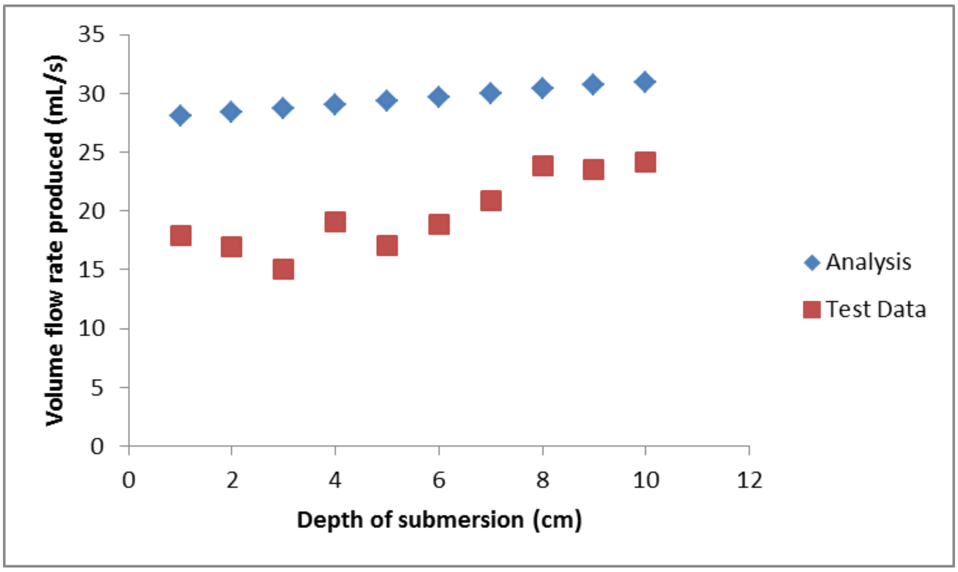

Figure 3: Volume flow rates in steady flow versus depth of submersion of the intake straw.

Unsteady experiments consisted of starting with the intake reservoir completely emptied of water and the intake straw being inserted as far into the intake reservoir as possible. Then, as little an amount of water was added into the reservoir to create the smallest depth of submersion achievable that still allowed the fountain to function properly. A test was run at that setting. Then, the depth of submersion of the short straw was increased in small increments and data were collected at each step along the way.

In the data shown below, ten changes of depths were tested. First, the tank was filled to a free-surface height of 1 cm, emptied, and the time it took to empty it was recorded; the tank was then filled to 2 cm, emptied, and the time taken was recorded as before. This process continued, using 1 cm increments, until a free-surface height of 10 cm was tested successfully. At each trial, the corresponding time it took to empty the tank was recorded. Thereafter, the corresponding flow rates were calculated by dividing each volume of fluid that was removed from the tank by the recorded time. The results for unsteady flows are plotted in Figure 4, where the dimensions of the straws used were D = 0.5cm, L_S = 21.2 cm, and L_L = 42.3 cm.

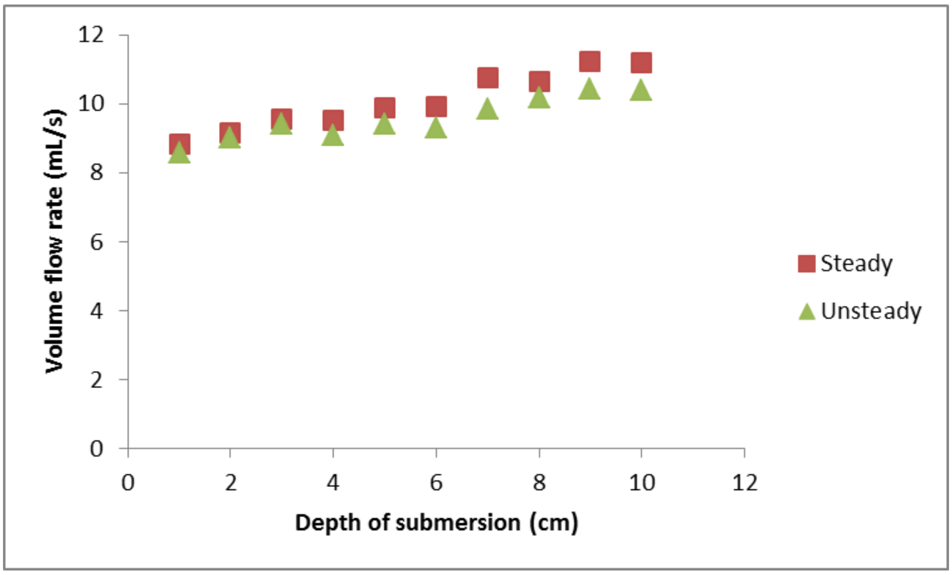

Figure 4: Depth of submersion versus flow rate for steady and unsteady flow.

CONCLUSIONS

Comparing the experimental results with those from analysis in Figure 3, one can see that there are discrepancies. On average, in steady flows, analytical flow rates were found to be 33% larger than those obtained experimentally. While these differences could come from a variety of sources, the major contribution is probably from losses that occurred in actual flows, but that were not fully accounted for in the analysis.

The calculated head losses were done as if water in the fountain had been conveyed from one reservoir to the other by means of one continuous straw. In reality, however, it was conveyed through two straws that were disconnected inside the upper chamber of the fountain. It was observed that when water exited the intake straw, it did so as a jet of water that spewed out of the tube, rose to a maximum height and, subsequently, splashed down onto the free surface of the water that was in the process of entering the exit straw. This splashing process produced bubbles most of which entered the exit pipe along with the outgoing water. This bubbly flow is not accounted for by the analysis presented in this course. These effects are the subject of experiments to be carried out by students in the future.

There were also discrepancies between the experimental results obtained from steady and unsteady flows. On average, flow rates from steady flows were 5% larger than those obtained when the flows were unsteady. The discrepancies were very small at the beginning of tests, but they were observed to increase with the flow rates. The major contribution was probably due to the downward acceleration of the free surface that occurred in unsteady flows. It appears to have introduced additional losses. This acceleration had not been accounted for in the analysis, because the variation of the velocity along the streamline was not known. This effect is also the subject of experiments to be carried out by students in the future.

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